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DEPARTMENT OF ELECTRICAL ENGINEERING
ACOUSTICS LABORATORY

EFFECTS OF ATMOSPHERIC TURBULENCE ON THE PROPAGATION OF SOUND

By
I. HORIUCHI

Technical Report No. 3
under
Office of Naval Research
Contract Nonr266(23)
Task No. NR 384-204

Submitted by
Cyril M. Harris
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CU3-54 — ONR266(23)-EE
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ABSTRACT

The principal objective of this report is to analyze existing information concerning the characteristics of atmospheric turbulence and with this as a basis to arrive at some representative figures on the resulting scattering of sound to be expected. This report first discusses the nature of such turbulence, the extent and causes of the anisotropy near the ground, the variation in its character with altitude, and the extent to which approximations of isotropy are valid. Other micrometeorological parameters are considered and their effects on sound propagation in comparison to velocity fluctuations estimated. Reference is made to available experimental data for which a reasonable spectral function for the turbulence has been constructed. This function is applied to a scattering cross section expression (8) to obtain numerical results for the scattering. These are compared with the few available experimental data.

1. Introduction

Of the manifold meteorological disturbances that affect the transmission of sound through the lowermost layers of the atmosphere, the turbulent motion of the air has been recognized for some time as playing a role of utmost significance. The characteristically severe fluctuations in received intensity of sound which results from it have been widely commented upon by numerous observers and some qualitative results obtained. Thus, for example, it is agreed that these fluctuations increase with the intensity of the turbulence, the frequency of the sound, and the separation between source and the receiver (1, 2). Yet quantitative records of such phenomena are singularly limited in number and detail. In the recent literature, papers by Sieg (3), Krasilnikov (4), and Richardson (5), and by Schilling and his associates on ultrasonics propagation (6), stand out as isolated examples.

The dearth of concrete data can hardly be ascribed to the lack of important applications since, apart from its intrinsic interest, such information is of value in studies of military sound ranging and aircraft noise propagation. The reason is to be found, as a reading of the literature readily makes apparent, in the difficult experimental procedures encountered in micrometeorological research. Indeed, as Schilling, et al (7) point out, any inquiry into the correlation between atmospheric sound propagation and local micrometeorological conditions immediately draws the investigator into the diverse and involved problems of the micrometeorologist. The instrumental demands are of the most exacting nature, e.g., the sensory elements employed (thermocouples, anemometers, etc.) must be capable of recording accurately fluctuations whose periods extend from milliseconds to many minutes. Careful interpretation of results is called for since instruments which measure the same parameter may, through differences in characteristics, yield data which are respectively difficult to compare.

The situation described above makes it difficult to prepare a set of consistent data in the form of tables or graphs which would allow quantitative predictions to be made on the amount of turbulent scattering. The solution obviously lies in improved instrumentation and further experimental work.

Theoretical expressions for the expected scattering in a turbulent medium have been derived by Kraichnan (8, 9). However, the application of these equations to atmospheric turbulence likewise requires definite information concerning its spectrum, and the above mentioned difficulty is encountered again.

The principal objective of this report is to analyze as much information concerning the characteristics of atmospheric turbulence as is presently available and with this as a basis to arrive at some representative figures on the scattering to be expected. Consequently this report will be divided into two parts, of which the first is a review of the available micrometeorological data pertinent to the study of sound propagation in open air. Such data will be derived solely from the available existing literature, since no new experimental work is to be reported upon in this paper. The second part consists of calculations in which expressions derived in the theoretical papers by Kraichnan (8, 9) will be used in conjunction with the above-mentioned data to indicate the scattering to be expected for the case of a medium isotropically turbulent, which represents conditions to be found in the upper regions of the atmospheric boundary layer and, for a certain range of eddy sizes, conditions found close to the ground. In view of the inexact character of the data employed, the figures obtained will apply strictly only to the particular case considered, although in order of magnitude, they will be correct for a wider range of data.

Before proceeding further, mention should be made of factors other than turbulence which contribute to the attenuation of airborne sound. The absorptive effect of humidity and the refractive bending of wave fronts due to the vertical stratification of wind velocity and temperature are well known and have been reported upon in a number of papers (2, 10, 11, 12). The latter two are responsible for the long period fluctuations which, depending upon the sign of the gradients involved, either cause or prevent the formation of "shadow zones". It is evident that the short period fluctuations on the order of seconds, which are the concern of this paper, do not involve these large scale vertical gradients, but only the high frequency departures therefrom. In addition to the attenuation due to water vapor in the air, the effects of viscosity and spherical divergence are also present. Though the first two show a rising characteristic with frequency, all three are of negligible importance here. Ground attenuation as a factor is also excluded from our consideration (12, 13).

2. Notation

In what follows, the customary notation $u_i (i=1,2,3)$ as denoting instantaneous wind velocities along the x, y, and z directions, respectively, will be used. Similarly, $x_i (i=1,2,3)$ will refer to the three space coordinates. On the assumption that most eddies are horizontal, which is frequently the case, if the direction of the mean wind velocity is chosen in the x_1-direction, cross-wind will be along the x_2 axis and vertically upward will be the x_3 axis. The mean wind velocity is defined by the expression $\bar{u}_1 = T^{-1}\int_0^T u_1\,dt$ where T is a suitably long time interval over which the average is taken. (It is acknowledged that the specification of such a mean value involves an arbitrary choice of T. To avoid a lengthy discussion, we shall adhere to customary practice, namely, the choice of T depends upon the periods of the fluctuations of interest.) According to the above orientation of the coordinate axes, $\bar{u}_2$ and $\bar{u}_3$ are zero. The instantaneous eddy velocities u_i' are defined as follows: u_i' is the difference between the instantaneous wind velocity u_1 and the mean wind velocity $\bar{u}_1$: $u_1' = u_1 - \bar{u}_1$; $u_2' = u_2$; $u_3' = u_3$.

The parameter which is correlated to the scattering of sound waves is the "gustiness" or "intensity of turbulence." This is identified by the symbol G, and is defined by $G_i = (\overline{u_i'^2})^{1/2}/\bar{u}_1$ This is the root mean square eddy velocity divided by the mean velocity.

3. Turbulence in the Atmosphere

3.1 Introduction

Atmospheric turbulence is essentially a large scale boundary layer problem in which, in contrast to the boundary layer depths encountered on the laboratory scale, on the order of centimeters, the depth of the layer is many hundreds of meters. This gives it the advantage of instrumental accessibility; on the other hand, the bounding surface displays irregularities never found in, say, a wind tunnel, a circumstance which greatly complicates the nature of the resulting turbulence. The main flow, essentially laminar, consists of the freely flowing winds in the atmosphere above the boundary layer, which are driven by pressure gradients and the Coriolis force. Upon coming into contact with the surface of the earth, these generate, through friction, turbulent eddies, and set up a vertical stratification of air motion in the lower atmosphere. These eddies are not to be thought of as simply circular vortices or waves but rather as fluctuations in the mean wind velocity, caused by moving masses of air with non-zero vorticity. As a series of similar eddies move past

Figure 1

EDDY SIZE AS RELATED TO MEAN WIND SPEED AND EDDY PERIOD

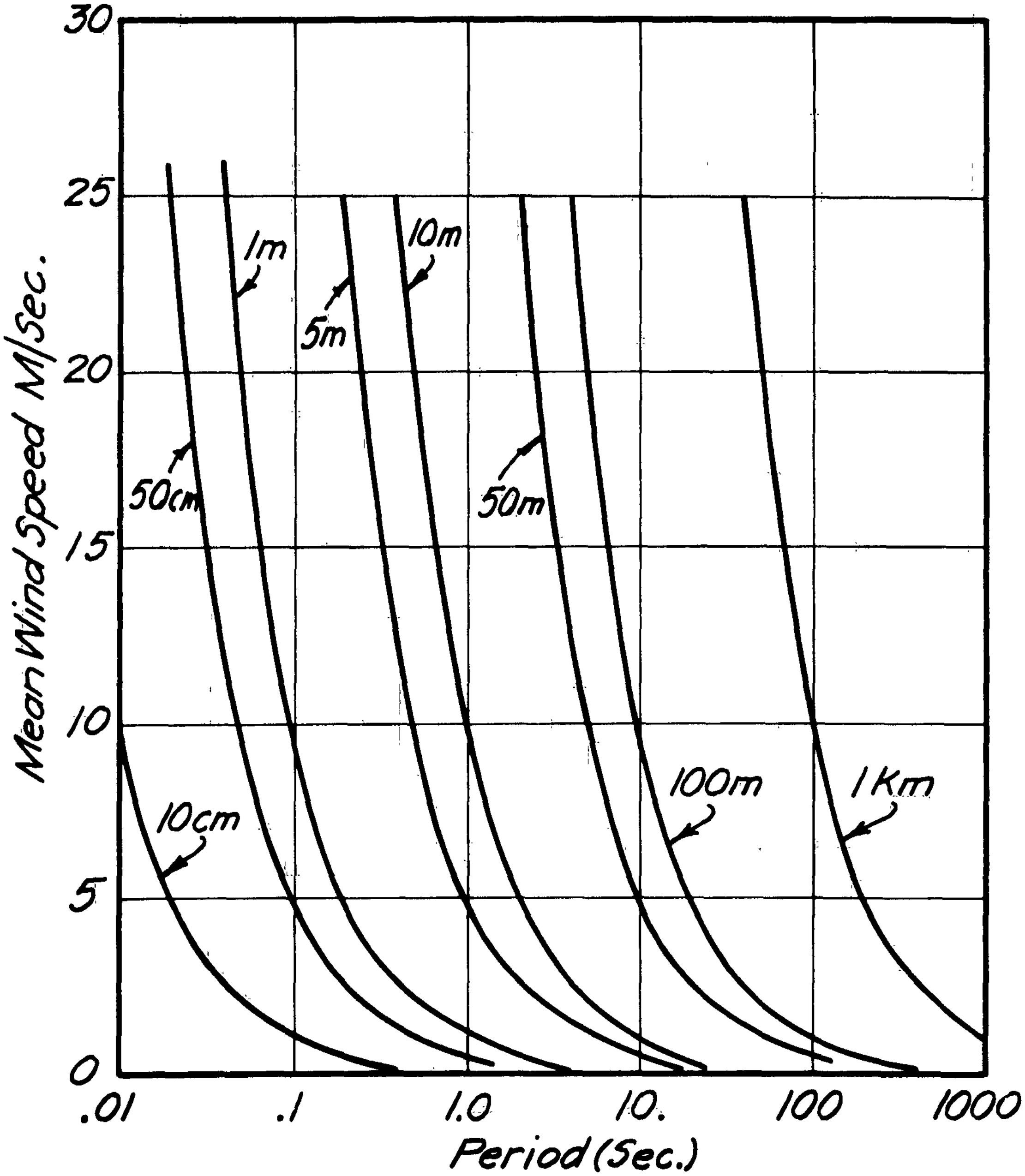
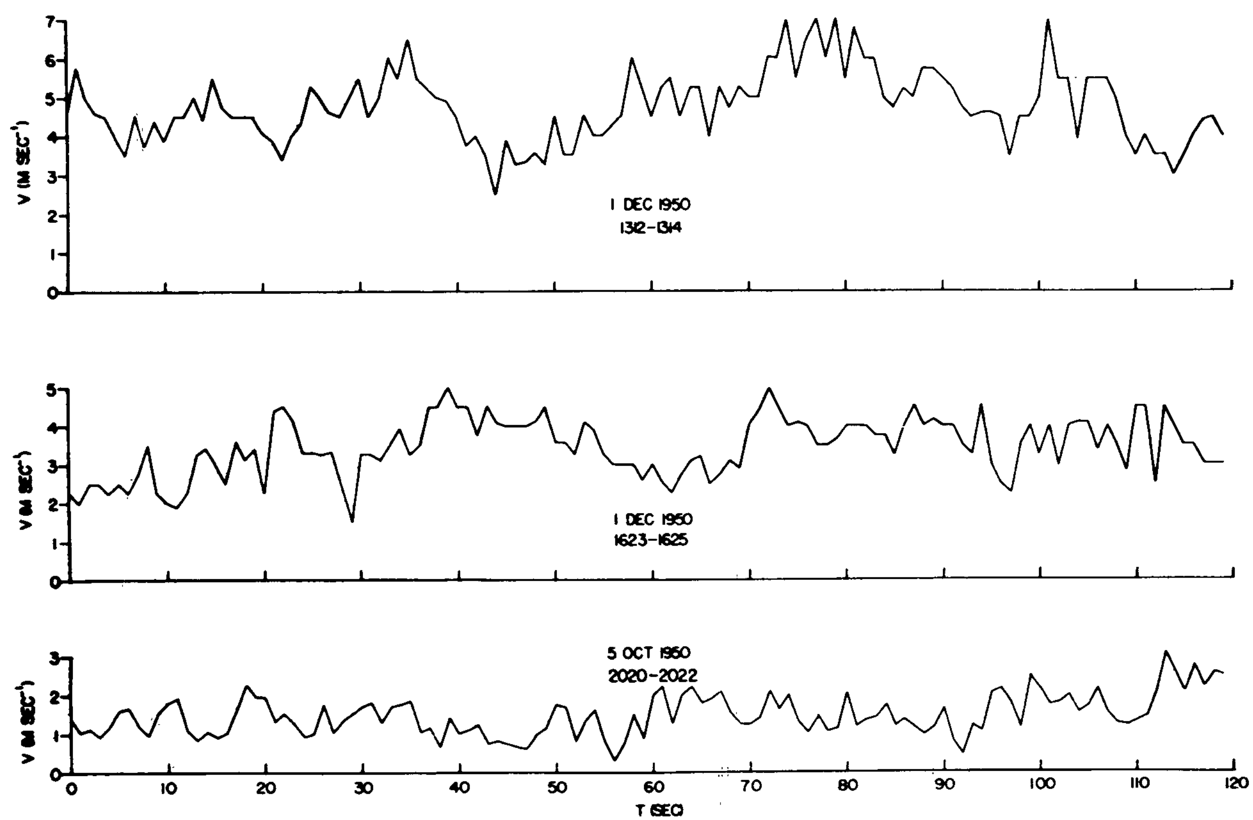


Fig. 2



Three examples of wind speed data from Cramer (I.S.A.T., p. 192) taken at a height of 2.5 m. This Figure shows mean wind speed plotted against time. The general tendency of the eddy fluctuations and the mean wind speed to decrease with time of day is evident. The fluctuations in wind direction also give similar graphs.

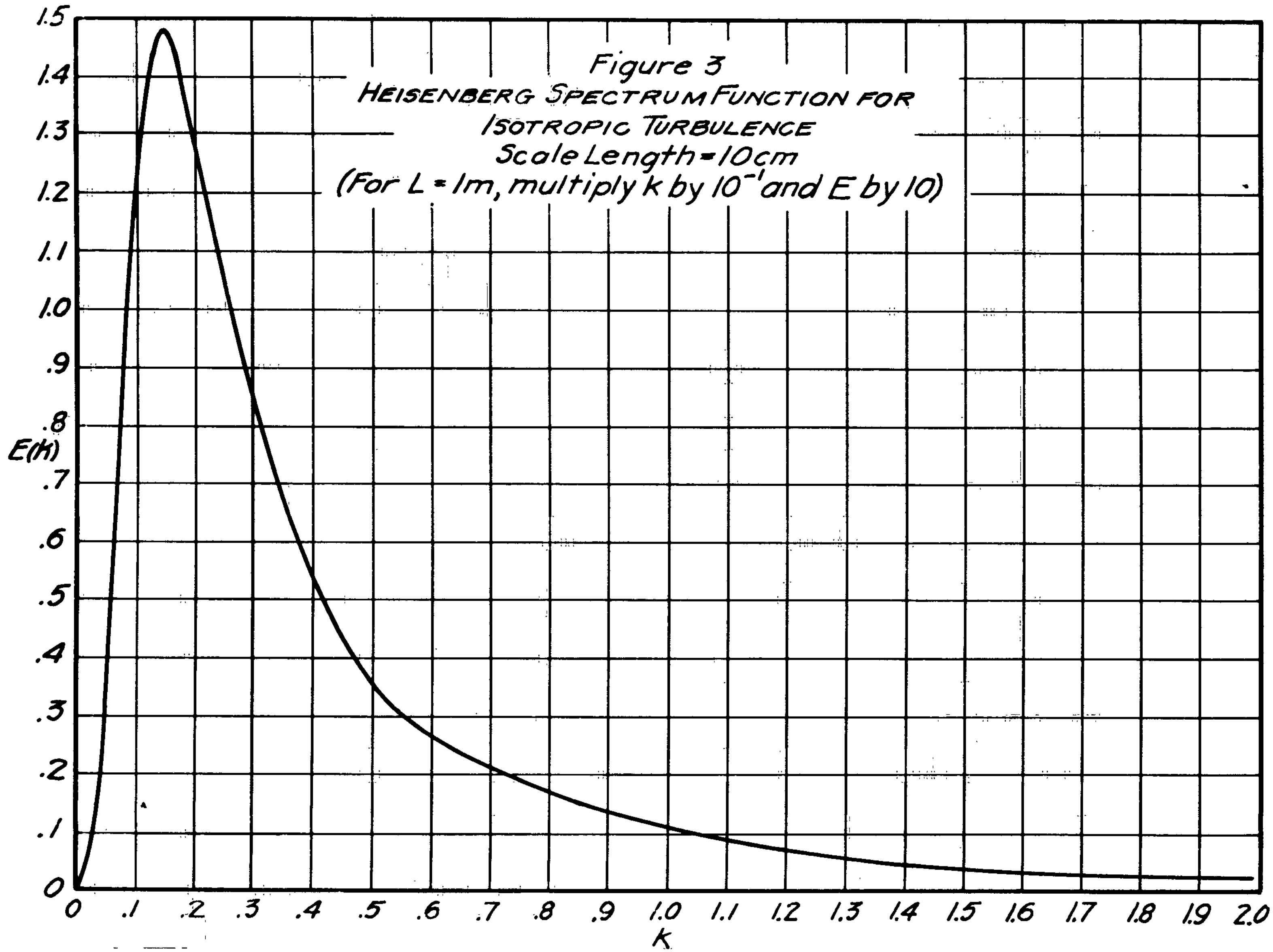


Figure 3
HEISENBERG SPECTRUM FUNCTION FOR
ISOTROPIC TURBULENCE
Scale Length = 10cm
(For L = 1m, multiply k by 10^{-1} and E by 10)

the observer, a periodicity in wind velocity and wind direction frequently will be observed. An estimate of the "eddy size" can be made if the period and the mean wind velocity are known (Figure 1). Usually, however, the velocity trace of a turbulent flow is quite irregular (Figure 2) and only rough estimates of size and period can be made, and for purposes of analysis, a Fourier decomposition is generally resorted to (Section 3).

The atmospheric boundary layer can be divided into two regions. The layer lowermost, the surface boundary layer, extends to a height of about 100 meters. Being immediately adjacent to the ground (or ocean), large inequalities are set up in the velocity fluctuation components along the three coordinate axes. Such anistropy of turbulent motion is quite characteristic of this layer. However, among those eddies whose size is small enough, of order $x_3/\bar{u}_1$, isotropy may locally exist. That is, the mean square velocity fluctuations are independent of the orientation of the coordinate axes: $\overline{u_1'^2} = \overline{u_2'^2} = \overline{u_3'^2}$. This is consistent with the observation that the eddy size increases with height so that isotropy of eddy motion extends to progressively larger eddy sizes as the elevation is increased. Above this layer, extending to a height of about 1 km, is the planetary boundary layer which shows little anistropy, the velocity fluctuations being fairly independent of direction.

It is evident from the above that the study of acoustic propagation involves the tracing of ray paths through either the surface layer alone, in which case the medium is predominantly anisotropic, or through both layers, the choice depending on the particular refractive conditions prevailing. In the interests of simplicity, however, we shall consider only the two pure cases of complete isotropy and anisotropy in this report.

The specification of conditions for isotropy is quite simple; consequently the main body of micrometeorological data to be considered will pertain to the departure from isotropy which is to be found in the surface boundary layer.

The random velocity fluctuations $u_i'(\vec{x},t)$ which characterize turbulent flow in the atmosphere are customarily assumed to arise from contributions to the flow by eddies of different sizes. Accordingly a Fourier decomposition of $u_i'(\vec{x},t)$ is resorted to:

$$u_i'(\vec{x},t) = (2\pi)^{-3/2}\int_{-\infty}^{+\infty} d\vec{k}\, e^{i\vec{k}\cdot\vec{x}}\, V_i'(\vec{k},t)$$

where $|\vec{k}| = 2\pi/\ell$ is the wave number vector belonging to the length ℓ , and v_i' is the Fourier amplitude. ℓ can be considered to be a kind of eddy size.

The values of u_i' at two separate points in space: $\vec{x}$ and $\vec{x}'$, will define a correlation tensor t_{ij} :

$$t_{ij}(\vec{x}, \vec{x}') = (\overline{u_i'^2})^{-1} \overline{u_i'(\vec{x},t)\, u_j'(\vec{x}',t)}$$

In the event the turbulence is isotropic, t_{ij} will be a function of the separation $\vec{\lambda} = \vec{x} - \vec{x}'$ only:

$$t(\vec{\lambda}) = (\overline{u_i'^2})^{-1} \overline{u'(\vec{x},t)\, u'(\vec{x}+\vec{\lambda},t)}$$

The Fourier transform of t_{ij} is given by:

$$S_{ij}(\vec{k}) = (2\pi)^{-3/2} \int_{-\infty}^{+\infty} t_{ij}(\vec{x})\, e^{i\vec{k}\cdot\vec{x}}\, d^3\vec{x}$$

and, under isotropic conditions, the energy density of turbulent fluctuations that lie within the limits k and $k + dk$ is:

$$E(k) = (2\pi)^{-1} \int_0^{\pi}\int_0^{2\pi} S_{ii}(k)\, k^2 \sin\theta \, d\theta \, d\phi$$

An idea of the average eddy size is obtainable by integrating $t(\lambda)$ as a function of λ. From the definition of the correlation function, $t(\lambda) = 1$ when $\lambda = 0$ and monotonically decreases to zero for some large value of λ. The more rapidly $t(\lambda)$ approaches zero, the smaller the eddy size. Therefore we shall define the average eddy size as

$$L = \int_0^{\infty} t(\lambda)\, d\lambda$$

3.2 Isotropic Turbulence

In the study of isotropic turbulence, it has been found convenient to look for situations where measurements are possible under controlled conditions. Advantage is taken of the fact that the assumption of isotropy reduces the significant parameters to two: gustiness and scale. One such opportunity exists inside wind tunnels where turbulent flow is produced by means of a fine mesh grid. At distances x on the order of $x/M \sim 10^2$ where M is the mesh size, the turbulence decays to an approximately isotropic state (14). It must be kept in mind, however, that conditions found in a wind tunnel differ from ground conditions in that there is no thermal stratification and that an upper limit to the

eddy size, which is determined by the mesh size used, exists. This tends to make the gustiness observed in wind tunnels smaller by a factor of five or more than that actually found close to the ground (15). In spite of this, the ease with which the above-mentioned parameters can be varied makes the wind tunnel an indispensable source of information in turbulence study.

The isotropic state as it exists in the wind tunnel is closely represented by the 3-dimensional Heisenberg function

$$E(k) = L^5 k^4 (1 + L^2 k^2)^{-3}$$

where L is the scale of turbulence defined previously. It is easily seen that for small wave numbers $E \propto k^4$ and for k large, $k \gg L^{-1}$; $E \propto k^{-2}$ (Fig. 3).

3.3 Effect of the Ground

The turbulent motion of the air which directly alters the character of sound propagation is found to be most intense within the first few meters of the ground. This is a consequence of the dominant role played by the ground in the formation of eddies (a) the ground is exposed each 24 hours to a heating-cooling cycle, the periodic temperature variations of which produce alternating positive and negative temperature gradients immediately adjacent to the surface; (b) irregularities in terrain features, by applying frictional drag to the prevailing air flow, generate turbulent eddies whose mean size is characteristic of the terrain. A shear flow is thus created with strong components parallel to the surface contours.

(a) Thermal Effects: The negative vertical temperature gradient $\partial T/\partial z < 0$ (normal lapse rate) which occurs during the day results in a convective upward transport of air which is superposed upon the prevailing wind and is maximum near the ground about noon. Accompanying this upward flow is usually an inflow of denser cool air which, upon striking the ground, produces short-period frictional eddies. Subsequently, the cool air is warmed by the ground and itself rises. It is apparent that this instability in density produces a large scale circulatory motion of the air in a vertical plane. This motion takes on a cell-like aspect, which has been studied by Bénard, and by Rayleigh and others, who conclude that the characteristic size of the eddies thus formed is of the order of 1 km in the direction of the wind and about half as much across. It is to be noted that the latter

figure exceeds by many times the wavelength of sounds normally encountered and hence that this type of eddy is not likely to be important in producing scattering. However, long period fluctuations in intensity commonly known as fading are ascribed to such large eddies. It may thus be concluded that the generation of frictional eddies by Bénard cells is, for this study, the most important result of the diurnal heating cycle. Parenthetically it should be added that the thermal stratification which the lapse rate represents is itself responsible for refractive effects in which distortion of the acoustic wave front occurs, especially in the vertical direction. As the lapse rate decreases with approach of night, there is a corresponding decrease in the gustiness. The transition from a normal lapse rate to inversion $\partial T/\partial z > 0$ that occurs as a result of rapid cooling of the ground causes a reversal of the density gradient and stabilizes the air motion. The large eddies referred to die out completely, with an accompanying reduction in the intensity of the frictional eddies. That the latter do not vanish altogether is due to the persistence of the mean flow. However, under extreme inversional conditions, it is found that even the frictional eddies are damped out. The fact that at night sounds are heard more clearly is due (aside from the general reduction in ambient noises) partly to this decrease in gustiness, although the more favorable refractive properties of the night air, which inhibits the formation of "shadow zones" is the predominant factor.

(b) Terrain Effects: The nature of the earth surface affects sound propagation by its role in the production of frictional eddies mentioned above, and by the absorptive and reflective role of the ground covering. The fluctuations in intensity and phase of interest, however, are due to the first factor. Since these frictional eddies arise from the flow of air over obstacles such as grass, trees, and buildings, it is natural to expect that vorticity not be oriented at random, but for the fluctuations in the direction of the wind to be greater than in other directions. This is shown in the next section to be the case. It is not easy to correlate the properties of the ground with the quantities commonly associated with turbulence, but according to Sutton (16), the gustiness should be related to the drag coefficient C_d. The accompanying table shows some representative

values for C_d. It can be seen that C_d is proportional to what one might consider the roughness of the surface.

Nature of Surface	C_d
Very smooth (mud flats, ice)	.002
Grass to 1 cm	.005
Thick grass to 10 cm high	.016
Thick grass to 50 cm high	.032
Sea	.002
Neutral equilibrium: $x_3 = 200$ cm $\bar{u}_1 = 500$ cm/sec.	

The role that the ocean plays in contributing to turbulent conditions above its surface is quite complicated since the roughness of the ocean is itself a function of the wind velocity.

The drag coefficient C_d for the ocean given in the table above ($C_d = .002$) applies only under the conditions stated of neutral equilibrium and wind speed of 5m sec^{-1} Observations seem to indicate that C_d is proportional to the wind speed and that for conditions other than neutral equilibrium, other values of C_d must be employed. The percentage turbulence over the ocean seems to either rise or remain constant with height, which indicates the persistence of turbulence to heights not found on land. There is evidence (25) to indicate that the height to which the turbulence extends above the ocean includes the tropopause, and perhaps into the stratosphere. In addition to the roughness due to shear motion of the air, the part played by the turbulent motion of the ocean itself in causing air turbulence is not known. There is evidently in this case very little information with which to work.

3.4 Anisotropy of Eddy Velocities

It is a well established fact that close to the ground, fluctuations in the vertical direction are not as great as in the horizontal. This difference, however, becomes less with increasing height (18). Such behavior is usually expressed meteorologically by the specification of the Reynolds shear stresses: $\tau_{ij} = -\rho\overline{u'_i u'_j}$ where ρ is the density. These stresses are indicative of momentum transport by the velocity fluctuations across the surfaces defined by i and j. A non-zero

value of $\tau_{ij}, i \neq j$ implies a correlation between u_i' and u_j' at that point, and therefore a preferential transfer of momentum. In particular, $\tau_{13} = -\rho \overline{u_1' u_3'}$ called the horizontal shear stress, measures the momentum transport across any plane parallel to the ground. Accordingly, the existence of a non-zero value of τ_{13} in a region, indicating correlation between horizontal and vertical eddy motion, is a condition for the presence of anisotropy. This situation has been observed in, and actually defines the extent of, the atmospheric boundary layer.

Observations on the relative difference between the various eddy components show that invariably u_2' exceeds u_3'. Scrase (19) reported that under conditions of small temperature gradient u_2' was larger than u_3' by 50% at a height of 2 meters with a gradual decrease to 20 meters, after which no asymmetry was found. This decrease has been questioned by several investigators (20) but no satisfactory explanation has been forthcoming. Best (21) found on the other hand, at a height of 200 meters, that G_2 exceeded G_3 by about 80%. This indicates that large variations in anisotropic conditions may be expected under varying terrain conditions. The importance attached to the ratio $u_2':u_3'$ lies in its effect upon the vertical and horizontal phase relationships in received sound which are especially significant in acoustic direction ranging. It is reported by Richardson (5) that phase fluctuations were noticeably greater when a source and receiver were set 6 ft. apart vertically than when set 12 ft. apart horizontally. A similar effect is also reported by Sieg (3).

A representative set of data illustrating the anisotropy is given in the accompanying table (22).

MEAN EDDY VELOCITIES AND SHEARING STRESS

Period (EST)	$\bar{u}_1$	$\bar{u}_2$	$\bar{u}_3$	$\bar{u}_1:\bar{u}_2:\bar{u}_3$	τ_{13}
1440-1447	140	88	32	1.0:0.63:0.37	4.5
1522-1529	122	83	50	1.0:0.68:0.41	4.8
1705-1712	103	70	51	1.0:0.68:0.50	4.3
1729-1736	92	60	56	1.0:0.65:0.61	5.0
2110-2117	45	25	15	1.0:0.56:0.33	0.2
2150-2157	63	36	23	1.0:0.57:0.37	1.1

Taken at height of 2.3 meters.

It can be seen that in general $\overline{u_1'} > \overline{u_2'} \geq \overline{u_3'}$ and that τ_{13} decreases with the change from normal lapse rate to inversion conditions. Data taken by other investigators show a similar range of values for τ_{13}, the differences being ascribed to mean wind speed, roughness of terrain, the height of observation, thermal stability of the atmosphere, and not least, the instrumentation employed.

As might be expected, the probability distribution of the eddy velocity amplitudes is Gaussian (21, 23). Each component roughly follows an equation of the form:

The gustiness, defined previously, is approximately independent of $\bar{u}_1$. It is shown (22, 23) that the velocity fluctuations increase in proportion to the mean wind, a dependence which seems to persist over all ranges of the lapse rate. Best (21) has found that G_1 is approximately constant for $2.5 < x_3 < 200$ cm, and is fairly independent of temperature in the range $2.5 < x_3 <$ 10 cm, with however a decrease as conditions change from lapse to inversion. Theory (24) indicates, on the other hand, that G_1 increases close to the ground, as conditions tend toward inversion. This seems to be supported by Cramer's data (22). There is evidently more data necessary before a definite conclusion is reached. Representative values of G_1 given in the literature range from 0.1 to 0.4.

3.5 Eddy Sizes

Observations by Scrase (19) and Panofsky (25) indicate that the average eddy size generally increases with altitude, which is reasonable in view of the decrease in frictional agents. The scale of turbulence L which is a rough measure of the average eddy size, owes its importance to the fact that the wave front of a compressional wave is distorted when it encounters a shear flow, the magnitude of the resulting scattering being determined by the size of the flow. We shall consider the longitudinal scale L_1 measured in the mean direction of wind and the transverse scale L_2 measured in a crosswind direction. It has been observed that L_1 is often longer than L_2 (22). As a matter of fact, there seem to exist daily fluctuations in both scales in which L_1 is larger than L_2 during the early afternoon and at night, while during the late afternoon they are approximately equal. At night both show a decrease in size (22). The representative values of L_1 taken at a height of 2.5 meters range from about 6m at night to about 8m in late afternoon whereas these of L_2 ranges from about 7m during the same period. This is indicative of an average eddy size of less than 10 meters and is in agreement with the scale found by Schilling

et al (6). With a mean daytime wind velocity of 5 m/sec., this would correspond to a mean eddy period of 1 to 2 sec. The reduction in mean wind velocity at night to about 1 m/sec increases the eddy period to about 6 sec. These values agree fairly well with those found by Scrase (19) and others.

It is to be noted that measurements of scale as calculated above depend very greatly upon the time interval over which the observations are taken. This of course is due to the increasing range of eddy sizes included. It has been found that as the time increases, the correlation coefficients rise asymptotically to a constant value. A representative value for the time in which such asymptotic values are reached by the longitudinal and transverse coefficients is about 100 seconds (22).

4. Calculations

In the paper by Kraichnan previously referred to (8), an explicit expression for the averaged total cross section for scattering $\langle\sigma\rangle$ for isotropic turbulence is given. According to this formula:

$$\langle\sigma\rangle = 16\pi^2\omega_0^4/c^6 \int_0^1 \langle E(\tfrac{2\omega_0 x}{c})\rangle\, x^2(1-2x^2)^2(1-x^2)^{3/2} \qquad (5.16)\ (8)$$

where σ is the total scattering cross section per cm^3, ω_0 is 2π times the incident frequency, c is the velocity of sound, x is $\sin\frac{1}{2}\theta$, θ is the angle between the incident wave and the scattered wave, and $E(2\omega_0 x/c)$ is the spectrum function for the kinetic energy per unit mass.

The three dimensional Heisenberg spectral density for isotropic turbulence previously given is:

$$E(k) = L^5 k^4/(1+L^2k^2)^3$$

where L is the scale of the turbulence and $k = \frac{\omega}{c}$ is the wave number of the flow. In order to employ this function in the expression for $\langle\sigma\rangle$, it is necessary to form the product $E(k)\frac{\overline{u'^2}}{k^2}$ where $\overline{u'^2}$ is the mean square of the velocity fluctuation components.

Since $k \sim \frac{2\omega_0 x}{c}$, the quantity $\langle E(\frac{2\omega_0 x}{c})\rangle$ is given by:

$$\langle E(\tfrac{2\omega_0 x}{c})\rangle = L^5 k^2\, \overline{u'^2}/(1+L^2k^2)^3 .$$

The complete expression for $\langle\sigma\rangle$ is then:

$$\langle\sigma\rangle = \frac{2\pi^2\omega_0^6\,\overline{u^2}\,L^5}{c^8}\int_0^1 x^2 G(x)\left(1+\frac{4\omega_0^2L^2}{c^2}x^2\right)^{-3}$$

where $G(x)$ is the quantity plotted in Figure 2, reference (8), and is defined as:

$$G(x) = 32x^2(1-2x^2)^2(1-x^2)^{1/2}$$

Since the principal scattering occurs for eddy sizes on the order of the incident wavelength λ_0, we may set $\lambda_0 \sim L$ so that $\omega/c = \frac{2\pi}{\lambda_0} \sim \frac{2\pi}{L}$. In which event, $\langle\sigma\rangle$ becomes

$$\langle\sigma\rangle = \frac{2^7\pi^8 f_0\,\overline{u^2}}{c^3}\int_0^1 \frac{x^2 G(x)}{(1+16\pi^2x^2)^3}\,dx$$

It is apparent that this expression indicates a direct dependence of the scattering upon the incident frequency f_0 and the gustiness, in agreement with experimental evidence. It will be recalled that the gustiness was previously defined as the root mean square velocity fluctuation $(\overline{u^2})^{1/2}$ divided by the mean velocity.

Numerical integration of the integral yields the value of 3.5×10^{-5}. With typical values of $f_0 = 100$ cycles, $\overline{u^2} = 1.5 \times 10^4$ cm^{2}sec^{-2} (23), and $c = 3.4 \times 10^4$ cm/sec, we get for $\langle\sigma\rangle$, 1.6×10^{-6} cm^{-1}. Employing the expression $I = I_o \exp(-\sigma x)$ where I is the intensity at distance x, and I^o is the initial intensity, calculation shows the loss per foot to be 0.21×10^{-3} db at 100 cycles.

The table below presents results from similar calculations for several frequencies. For comparison, experimental results recently obtained by Delsasso (27) in the high Sierras under varying atmospheric conditions are shown alongside. The elevations at which these data were taken, about 500 m. maximum, are high enough to assume that isotropy exists for the eddy sizes under consideration. These values are not strictly comparable, however, since the attenuation due to diffraction and humidity is also included.

	Results	
frequency	calculated	observed (Delsasso,27)
100 ~	0.21×10^{-3}	.012 to 1.12×10^{-3}
250	0.53	.36 1.47
500	1.05	1.06 2.64
1000	2.10	.83 2.56

It is apparent that the expression from which the above results were calculated yields results of the correct order of magnitude.

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NOTE: I.S.A.T. refers to the publication International Symposium on Atmospheric Turbulence in the Boundary Layer, Geophysical Research Papers #19, M.I.T. (1951).

J.A.S.A. refers to Journal of the Acoustical Society of America.